RANDOM VARIABLES & DISTRIBUTION FUNCTIONS

Probability density Function & Distribution function:

A variable which can assume only a countable number of real values & for which the value which the variable takes depends on chance, is called a discrete random variable.

The probability of an event when expressed as a function is called p.d.f. e.g. the binomial $^nC_r p^r q^{n-r}$ & the poisons distribution function $(e^{-m}m^r)/r!$

A feature of a p.d.f. $\Sigma f(x)$ for all x =1 if x is a discrete random variable & $\int f(x)dx$ for all x=1 if x is a continuous random variable.

(1) The p.d.f. of a random variable X is given by f(x)=x/6, for x=1, 2, 3.
  a. Write the probability distribution of X & draw a histogram
  b. Find P(X<3)

(2) The p.d.f. of a random variable X is given by, f(x)=c(x+2) for x=0, 1, 2, 3, 4. Find the value of c & hence P(X>1)

(3) The probability density function of a random variable X is given by $f(x)=(1/5)(4/5)^x$ for x=0,1,2,3,…… find P(X≥3).

(4) A random variable X has the following probability function:

Values of X,x:	0	1	2	3	4	5	6	7
p(x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

  (i) Find k
  (ii) Evaluate P(X<6), P(X≥6) and P(0<X<5)
  (iii) If P(X≤a)>1/2, find the minimum value of a, and
  (iv) Determine the distribution function of X.

☺[NOTE: distribution function F(a)=Σf(x) where 0≤x≤a]

(5) If, p(x) = x/15; x=1,2,3,4,5
  = 0; elsewhere

  Find (i) P{X=1or2}, and (ii)P{1/2<X<5/2 | X>1}

☺[NOTE: P(A|B)=P(A∩B)/P(B)]

(6) Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces, Construct a table giving the non-zero values of the probability mass function & draw the probability chart. Also find the distribution function of X.

(7) An experiment consists of three independent tosses of a fair coin. Let : X=the number of heads, Y=the number of head runs, Z=the length of head runs, a head run being defined as consecutive occurrences of at least two heads, its length then being the number of heads occurring together in three tosses of the coin.
  Find the probability function of (i)X, (ii)Y, (iii)Z, (iv)X+Y & (v)XY and construct probability tables & draw their probability charts.

(8) The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f: $f(x)=6x(1-x), 0\le x\le 1$

a. Check that f(x) is p.d.f, and
b. Determine a number b such that $P(X<b)=P(X>b)$

(9) A continuous random variable X has a p.d.f. $f(x)=3x^2, 0 \le x \le 1$. Find a & b such that

a. $P(X\le a)=P(X>a)$
b. $P(X>b)=0.05$

(10) Let X be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} ax & , 0 \le x \le 1 \\ a & , 1 \le x \le 2 \\ -ax+3a & , 2 \le x \le 3 \\ 0 & , \text{elsewhere} \end{cases}$$

a. Determine the constant a
b. Compute $P(X\le 1.5)$

(11) The probability distribution of a r.v. X is $f(x)=k\sin\frac{\pi x}{5}, 0\le x\le 5$. Determine the constant k and obtain the median & quartiles of the distribution.

(12) A random variable X is distributed at random between the values 0 and 1 so that its probability density function is :$f(x)=kx^2(1-x^3)$, where k is a constant. Find the value of k. Using this k, find its mean & variance.

(13) A variable X is distributed at random between the values 0 & 4 & its probability density function is given by: $f(x)=kx^3(4-x)^2$. Find the value of k, the mean and standard deviation of the distribution.

(14) Calculate the standard deviation and mean deviation from mean if the frequency function f(x) has the form:

$$f(x)= \frac{3+2x}{18}, \text{ for } 2\le x \le 4$$
$$= 0 \quad \text{otherwise}$$

(15) Show that for the symmetrical distribution:

$$f(x) = \frac{2a}{\pi}\left(\frac{1}{a^2+x^2}\right), -a\le x \le a$$

$$\mu_2=\frac{a^2(4-\pi)}{\pi} \text{ and } \mu_4=a^4\left(1-\frac{8}{3\pi}\right)$$

(16) Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with p.d.f. given by

$$f(x) = \frac{100}{x^2}, \quad \text{when } x \geq 100$$
$$= 0, \quad \text{elsewhere}$$

a. What is the probability that all of three such tubes in a given radio set will have to be replaced during first 150 hours of operation?
b. What is the probability that none of the three of the original tubes will have to be replaced during that first 150 hours of operation?
c. What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?
d. What is the maximum number of tubes that may be inserted into a set so that there is a probability of 0.5 that after 150 hours of service all of them are still functioning?

(17) The time one has to wait for a bus at a downtown bus stop is observed to be random phenomenon (X) with following probability density function:

$$f(x) = \begin{cases} 0, & \text{for } x<0 \\ (x+1)/9, & \text{for } 0 \leq x < 1 \\ 4(x - 0.5)/9, & \text{for } 1 \leq x < 3/2 \\ 4(5/2 - x)/9, & \text{for } 3/2 \leq x < 2 \\ (4 - x)/9, & \text{for } 2 \leq x < 3 \\ 1/9, & \text{for } 3 \leq x < 6 \\ 0, & \text{for } x \geq 6 \end{cases}$$

Let the events A and B be defined as follows:

A:One waits between 0 to 2 minutes inclusive

B: One waits between 1 to 3 minutes inclusive

a. Draw the graph of probability density function
b. Show that
   i. $P(B|A) = 2/3$
   ii. $P(\bar{A} \cap \bar{B}) = 1/3$

(18) The amount of bread (in hundred of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the p.d.f. f(x), given by:

$$f(x) = \begin{cases} kx, & \text{for } 0 \leq x < 5 \\ k(10 - x), & \text{for } 5 \leq x < 10 \\ 0, & \text{otherwise} \end{cases}$$

a. Find the value of k such that f(x) is a probability density function.
b. What is the probability that the number of pounds of bread that will be sold tomorrow is:

i. More than 500 pounds
ii. Less than 500 pounds
iii. Between 250 & 750 pounds?

c. Denoting A, B and C the events that the pounds of bread sold are as in b (i), b(ii), b(iii) respectively, find P(A|B), P(A|C). Are
   i. A and B independent events
   ii. Are A and C independent events

(19) The kms X in thousands of kms which car owners get with a certain kind of tyre is a random variable having probability density function:

$$f(x) = (1/20)e^{-x/20}, \quad \text{for } x > 0$$
$$= 0, \quad \text{for } x \leq 0$$

Find the probability that one of these tyres will last

a. At most 10,000 kms.
b. Anywhere from 16,000 to 24,000 kms.
c. At least 30,000 kms.

(20) Verify that the following is a distribution function:

$$F(x) = 0, \quad x < -a$$
$$= 0.5\,(x/a + 1), \quad -a \leq x \leq a$$
$$= 1, \quad x > a$$

(21) The diameter, say X, of an electric cable, is assumed to be a continuous random variable withh p.d.f. $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$

a. Check the above is a p.d.f.
b. Obtain an expression for the c.d.f. of X,
c. Compute $P(X \leq 0.5 \mid 1/3 \leq X \leq 2/3)$
d. Determine the number k such that $P(X<k) = P(X>k)$

(22) Let X be a continuous random variable with p.d.f. given by:

$$f(x) = kx \quad , 0 \leq x < 1$$
$$= k \quad , 1 \leq x < 2$$
$$= -kx + 3k \quad , 2 \leq x < 3$$
$$= 0 \quad , \text{elsewhere}$$

a. Determine the constant k
b. Determine F(x), the c.d.f.
c. If x_1, x_2, x_3 are three independent observations from X, what is the probability that exactly one of these three numbers is larger than 1.5?

(23) A petrol pump is supplied with petrol once a day. If its daily volume of sales (X) is thousands of litres in distribution is given by: $f(x) = 5(1-x)^4$, $0 \leq x \leq 1$, what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01?

(24) A bombing plane carrying three bombs flies directly above a railroad track. If a bomb falls within 40m of track, the track will be sufficiently damaged to disrupt the traffic. With a certain bomb site the points of impact of a bomb have the probability density function:

$$f(x) = \begin{cases} (100+x)/10{,}000, & \text{when } -100 \le x < 0 \\ (100-x)/10{,}000, & \text{when } 0 \le x < 100 \\ 0, & \text{elsewhere} \end{cases}$$

where x represents the vertical deviation (in metres) from the aiming point, which is the track in this case. Find the distribution function. If all the three bombs are used, what is the probability that the track will be damaged?

(25) Suppose that the time in minutes that a person has to wait at a certain bus stop is found to be a random phenomenon, with a probability function specified by the distribution function:

$$F(x) = \begin{cases} 0, & x < 0 \\ x/8, & 0 \le x < 2 \\ x^2/16, & 2 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

a. Is the distribution function continuous? If so, give the formula for its probability density function
b. What is the probability that a person will have to wait
   i. More than 2 minutes
   ii. Less that 2 minutes
   iii. Between 1 & 2 minutes
c. What is the conditional probability that the person will have to wait for a bus for
   i. More that 2 minutes, given that its is more that 1 minute
   ii. Less than 2 minutes given that it is more than 1 miniute